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Preface

A detailed understanding of flows in thin liquid films is important for a wide range of modern engineering processes. This is particularly so in chemical and process engineering, where thin liquid films are encountered in heat-and-mass-transfer devices (*e.g.* distillation columns and spinning-disk reactors), and in coating processes (*e.g.* spin coating, blade coating, spray painting and rotational moulding). In order to design these processes for safe and efficient operation it is important to build mathematical models that can predict their performance, to have confidence in the predictions of the models, and to be able to use the models to optimise the design and operation of the devices involved.

Thin liquid films also occur in a variety of biological contexts, including the thin liquid linings of the airways in the lung and the thin tear films that coat the eyes, and greater understanding of these films should lead to improved treatment of diseases in these organs.

When liquids flow in thin films, the interface between the liquid and surrounding gas can adopt a rich variety of interesting waveforms. These shapes are determined by a balance of the principle driving forces, usually including gravity, surface-tension and viscous effects. The influence of other effects, such as rotation, inertia, inhomogeneity, non-Newtonian behaviour or Marangoni phenomena arising from the variation of surface-tension with temperature or with concentration of surfactants, can add significantly to the complexity of the observed phenomena. An excellent review of much of the early work on thin liquid films is given by Oron, *et al.* [1].

In recent years, there have been significant advances in the computational and analytical techniques for studying thin-film flows. These are often combined with more traditional asymptotic methods and normal-mode stability analysis that have been given a new lease of life by the wide availability of good computing resources. In this special double issue, we have gathered papers that demonstrate the present state of the art in describing thin-film flows and illustrate the broad range of their application.

Acrivos and Jin address the stability of rimming flow in which a thin film of liquid flows on the inside of a cylinder that rotates about its horizontal axis. They present a model equation whose form is based on asymptotic analysis, but whose range of validity appears to be much greater than might have been expected. The benefit of the model equation is that it includes the effect of gravity in producing a puddle of liquid at the base of the cylinder when the rotation is small or absent, and so extends the classical theory to a much wider range of filling fractions. The equation admits "inhomogeneous" asymmetrical solutions with localised rapid adjustments of the film thickness, in accordance with experimental observations. These inhomogeneous solutions are found to be stable, whereas the more benign-looking homogeneous solutions are found to be neutrally stable; inclusion of weak surface-tension effects then confirms that the homogeneous solutions are susceptible to a Rayleigh-type instability of long wavelength in the transverse direction.

Over the last 40 years the nonlinear dynamics of a thin liquid film flowing down an inclined plane have been extensively studied using the Benney equation. Oron and Gottlieb revisit the problem of the stability threshold predicted by this equation and deduce that, whereas the primary bifurcation of the first-order Benney equation is supercritical for small values of the Reynolds number but subcritical for larger values, the primary bifurcation of the second-order Benney equation is always supercritical.

Edmonstone *et al.* address the fingering instability at the leading edge of a thin liquid film flowing down an inclined plane. They focus on Marangoni effects due to the presence of surfactants on the free surface of the film and first demonstrate the complex profile adopted at the film front in the two-dimensional case. A capillary ridge forms at the front, but is preceded by a "step", and followed by a dip and a slow decline in thickness, all of which are attributed to Marangoni effects. Here the basic flow is time-dependent, so a transient growth analysis is required to assess the stability of the flow to perturbations that are periodic in the transverse direction. The "energy" of these modes is investigated and the authors show that an instability of moderate wavelength targets the leading edge of the front, just behind the Marangoni step.

Marangoni effects due to the presence of surfactants are also the subject of the paper by Schwartz *et al.* who formulate and solve numerically a mathematical model for the surfactant-driven motion of liquid droplets. In particular, the authors use their model to show that it is possible to split one droplet into two by releasing surfactant into the free surface appropriately, and conjecture that their results are relevant to the basic mechanisms involved in biological cell division.

Marangoni effects due to variations of surface-tension with temperature rather than surfactant concentration are also the subject of the work by Trevelyan and Kalliadasis who study the draining of a thin liquid film down a uniformly heated wall. In particular, the authors rectify the deficiencies in earlier models, and use their new models to construct improved bifurcation diagrams for solitary waves which are compared to those predicted by the earlier models. Numerical computations of the evolution of the layer show that both the free-surface shape and the interfacial temperature distribution approach a train of coherent structures that resemble the infinite-domain stationary solitary waves predicted by the bifurcation analysis.

Diverse new physical effects can be incorporated within the mathematical restrictions of standard lubrication theory, unveiling interesting new behavioural regimes and instabilities. Sultan *et al.* consider the interaction of a thin film of a polar liquid on a solid substrate with an overlying semi-infinite region of gas into which the liquid is evaporating. The evaporation regime is taken to be diffusion-limited so that the mathematical model is a standard lubrication-type evolution equation for the layer height coupled with a non-local interaction term deriving from the solution of Laplace's equation in the upper layer. The linear and weakly-nonlinear stability properties of a simple base-state solution to this problem are presented, in which evaporation and surface-tension interact with a disjoining pressure associated with electrostatic and van der Waals effects. In their paper, Papageorgiou and Petropoulos also study the effects of electric fields on the linear stability of charged liquids. These authors study the situation of a flat liquid layer surrounded by dynamically-inactive gas both above and below it. It is assumed, however, that electric-field effects are present both above and below the layer, leading, once again, to interface-induced interactions of a Laplacian field (in this case, for the electric-field potentials) with the fluid dynamics of the layer.

The extension of the methods of standard lubrication theory to non-Newtonian fluids is an interesting and challenging one. Flitton and King investigate the dynamics of thin layers of both Newtonian and power-law fluids in the case where a contact line exists between the fluid layer and a substrate that is being dewetted under the effects of surface-tension. This paper is a particularly good showcase for the power of asymptotic methods in studying thin-film models.

Beyond the incorporation of different and more varied physical effects, there remain many mathematical challenges in the field of thin-film flows. The overriding mathematical advantage of thin-film theories is that they take advantage of a wide separation of scales in the geometrical configuration under consideration. This affords valuable simplification, obviating the need for computationally-expensive fully numerical simulations while preserving essential elements of the physics of the system. For example, the classical lubrication approach provides a uniformly valid model when the variations of the curvature of a substrate and any thin film placed on it are large compared to the film depth. Such conditions signally fail to be satisfied, for example, at any sharp corners in the substrate. Two papers in this double issue address the interesting question of whether the inherent advantages of thin-film models can in any way be salvaged, given this apparently devastating circumstance. Motivated by the problem of thinning Plateau borders in the evolution of foams, Stocker and Hosoi present what might be termed a "non-traditional" generalization of thin-layer analysis in which they introduce hyperbolic coordinates to derive generalized thin-layer equations where the "thin-ness" of the layer is in the direction of a hyperbolic coordinate following the shape of a 90-degree corner. With this new formulation, they succeed in showing that certain of the advantages of thin-layer theory do indeed survive. With generally similar intentions, Jensen *et al.* study thin-film flows near isolated humps and corners as arise, for example, when geometrical imperfections create defects in coating flows. These authors adopt a rather different approach to Stocker and Hosoi and tackle the geometrical complications using modified thin-film equations associated with a reparametrization of the layer description using spines.

Advances in general numerical techniques are also important, and an example included here is the paper by Pozrikidis which describes a robust computational approach for flows with deformable interfaces. The immersed-interface method is combined with a diffuse-interface approximation, where the interfacial region is represented by a continuous variation of fluid properties and a distributed surface-tension force. The method is then applied to study the combined effects of inertia and Marangoni stresses on the stability of a two-layer channel flow. The companion paper by Blyth and Pozrikidis presents a normal-mode linear stability analysis of the flow and shows that the numerical method is capable of matching the linear results and extending them well into the nonlinear regime, even beyond the point of wave overturning.

The 12 papers in this special double issue demonstrate the state of the art in describing thin-film flows, and illustrate both the wide variety of mathematical methods that have been employed and the broad range of their application. Despite the significant advances that have been made in recent years there are still many challenges to be tackled and unsolved problems to be addressed, and we anticipate that thin liquid films will be a lively and active research area for many years to come.

References

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D.G. Crowdy
Department of Mathematics, South Kensington Campus,
Imperial College, London SW7 2AZ, United Kingdom
C.J. Lawrence
Department of Chemical Engineering, South Kensington Campus,
Imperial College, London SW7 2AZ, United Kingdom
S.K. Wilson
Department of Mathematics, University of Strathclyde, Livingstone Tower,
26 Richmond Street, Glasgow G1 1XH, United Kingdom